COMPUTATION OF ELECTRIC-ARC PARAMETERS TAKING ACCOUNT OF CONVECTIVE AND RADIANT LOSSES

N. A. Zyrichev

UDC 537.523:536.3:536.25

An approximate solution of the problem of an electrical arc in a turbulent argon stream is obtained, taking into account convective and radiant energy losses in the discharge channel at atmospheric pressure.

Electric-arc heaters with arc stabilization by a gas stream in long cylindrical channels are presently being used in various branches of science and technology. The Reynolds numbers determined by the stream parameters at the entrance to the discharge channel are $\sim 10^4$. Moreover, the stream of working gas entering the discharge channel usually possesses an initial noticeable turbulence, and hence the gas flow in real heaters may be turbulent. Belyanin [1] proposed a turbulent arc model for an approximate estimate of the basic heater parameters, and it received further development and extension in [2-4]. The turbulent model of a longitudinally cooled arc is based on the analogy between the development of an arc discharge in a gas stream and the propagation of a nonisothermal jet in a co-flow. The heat exchange between the strongly heated central zone, in which the Joulean heat of the arc is evolved, and the cold peripheral gas layers is considered to be determined by turbulent mixing. The abundant empirical material accumulated in investigations of nonisothermal jets [5] is used for a quantitative description of the fundamental processes occurring in the discharge channel.

Within the scope of the turbulent model, the problem of the distribution of the stream and discharge parameters in a cylindrical channel has been solved approximately taking account of convective heat exchange between the gas and the wall [4]. A solution of this same problem is presented below, taking account of the radiant energy losses, for the case of an arc in an argon stream at atmospheric pressure. We shall not formulate the problem since it is described in detail in [4]. The change in the system of equations of the problem only concerns the energy equation in which the energy flux due to volume radiation of the plasma is introduced:

$$i\frac{dV}{dx} = \frac{d}{dx}\left(\int_0^R 2\pi\rho uhrdr\right) + 2\pi Rq_w + \int_0^R 2\pi q_r rdr. \tag{1}$$

Integrating (1) and going over to dimensionless quantities, we obtain the energy equation in the following form (taking account of the simplifications introduced in [4]):

$$IU = \Delta\bar{h}_a\left[1 + 0.58\,\mathrm{Re}_{wd}^{-0.2}H(\xi)\right] + \int_0^{\bar{x}} \xi^{-2}\int_0^{\xi}\frac{\pi q_r d^3}{4Gh_0}\,\eta d\eta d\bar{x}. \tag{2}$$

Using the equation for propagation of a turbulent nonisothermal jet in a co-flow [4]

$$\frac{d\xi}{d\bar{x}} = -0.1\xi^2\Delta\bar{h}_m^{0.5}, \tag{3}$$

we convert the last number of the energy equation to

State Institute of the Nitrogen Industry, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 4, pp. 673-680, October, 1971. Original article submitted July 13, 1970.

$$\int_0^{\bar{x}} \xi^{-2} \int_0^{\xi} \frac{\pi q_r d^3}{4Gh_0} \eta d\eta d\bar{x} = 3.925\, Q \int_{\xi}^{\infty} \int_0^{\xi} \frac{q_r}{q_0} 2\, \eta d\eta \frac{d\xi}{\xi^4\, \Delta \bar{h}_m^{0.5}} \cdot$$

Here Q is a dimensionless criterion characterizing the intensity of volume radiation under the considered conditions, and q_0, W/m^3, is the characteristic magnitude of the specific energy flux due to the volume radiation. As we see, the fraction of the radiation in the energy balance of the arc is determined by the gas pressure and species (this is taken into account in terms of q_0), and the gas discharge G, and depends quite strongly on the diameter of the discharge channel.

Radiation is ordinarily neglected in arc computations at almost atmospheric pressure. At the same time, computations [6] and experiments [7] show that an arc in an argon stream loses 10-25% of its power because of radiation even at atmospheric pressure. The available literature data on the magnitude of the volume radiation at atmospheric pressure for the most widespread working gases, such as argon, air, nitrogen, and hydrogen, have been generalized in [8]. The temperature dependence of the energy flux q_r emitted by unit volume is quite complex and has a maximum in the $(16\text{-}18)\cdot 10^3$ °K range. In order to obtain an analytical solution which would permit an approximate estimation of the influence of radiation on the arc characteristics without large expenditures of time, we must make a number of simplifications.

Let us write

$$\int_0^{\xi} 2q_r \eta d\eta = \int_0^{1} 2q_r \eta d\eta y(\xi,\ \Delta\bar{h}_m) = w\,(\Delta\bar{h}_m)\, y\,(\xi,\ \Delta\bar{h}_m).$$

At constant pressure q_r depends only on the enthalpy, and since the enthalpy profile in the mixing zone is given, the function w can be evaluated for any value of $\Delta\bar{h}_m$. Let us assume that the function $y(\xi, \Delta\bar{h}_m)$ depends only on ξ and on the basis of a computation let us introduce an approximation for argon

$$y = \begin{cases} 1 & \text{for} \quad \xi \geqslant 0.57; \\ 2.32\xi^{1.5} & \text{for} \quad \xi < 0.57; \end{cases}$$

$$w = q_0 \Delta\bar{h}_m^{1.5}, \quad q_0 = 8\cdot 10^4\, \mathrm{W/m^3}.$$

The results of computing the function $w(\Delta\bar{h}_m)$ for argon are represented in Fig. 1 by open circles, and the approximation for w by curve 1.

Radiation contributes to equilibration of the gas enthalpy on the arc axis. Hence, we assume the quantity $\Delta\bar{h}_m$ to be constant in evaluating the integral in the right side of (2), and take it outside the integral sign. As a result of the assumptions made, the energy equation becomes a linear dependence between the quantities U and $\Delta\bar{h}_m$

$$IU = \Delta\bar{h}_m \left[\Phi(\xi) + 0.58\, \mathrm{Re}_{wd}^{-0.2} H(\xi)\, \Phi(\xi) + 3.925 Q W(\xi)\right] = \Delta\bar{h}_m A(\xi). \tag{4}$$

The auxiliary functions are defined by the formulas

$$\Phi(\xi) = \xi^{-2} \int_0^{\xi} 2f(\xi)\, \xi d\xi;$$

$$H(\xi) = \int_{\xi}^{1} [f(\xi)]^{1.25}\, [\Phi(\xi)]^{-0.75} \xi^{-2} d\xi;$$

$$W(\xi) = \int_{\xi}^{\infty} y(\xi)\, \xi^{-4} d\xi;$$

$$f(\xi) = (1 - \xi^{1.5})^2; \ \xi \leqslant 1.$$

Let us convert the equation for the total current into

$$i = \int_0^{R} 2\pi\sigma \frac{dV}{dx} r dr = \frac{dV}{dx} \pi b^2 \int_0^{\xi} 2\sigma \eta d\eta = \xi^{-2} \pi R^2 \sigma_a \frac{dV}{dx}.$$

The total conductivity of an arc column σ_a depends on $\Delta\bar{h}_m$ and the coordinate ξ. Let us assume that

$$\sigma_a = \varphi(\Delta\bar{h}_m)\, \Psi(\xi).$$

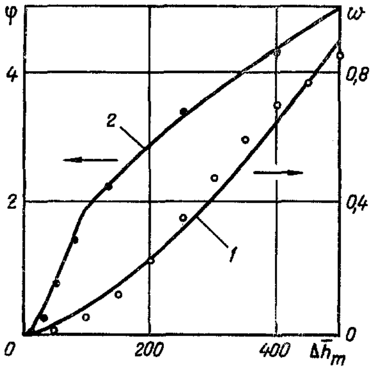

Fig. 1. The functions $\varphi(\Delta\bar{h}_m)$ and $w(\Delta\bar{h}_m)$ for argon at atmospheric pressure (φ, 10^3 ohm^{-1} $\cdot$ m^{-1}; w, 10^9 W/m^3).

The computed values of the function φ for argon at atmospheric pressure are represented in Fig. 1 (filled circles). To obtain an analytical solution of this problem, it is desirable to approximate this dependence by a power-law function. Since the character of the change in φ for small and large values of $\Delta\bar{h}_m$ is essentially distinct, the approximation of φ by one smooth function is hardly justified. It is known that a change in the slope of the volt–ampere characteristics of an arc in a gas stream is related directly to the change in the character of the rise in electrical conductivity during the passage from low and moderate to high temperatures. In order to reflect this in the solution, it is necessary to approximate the function φ by a piecewise-smooth power-law function

$$\varphi = \sigma_0 \Delta\bar{h}_m^{\alpha},$$

where the quantities σ_0 and α have a step-like change at some critical value of the dimensionless excess gas enthalpy on the arc axis $(\Delta\bar{h}_m)_{cr}$. As is seen from Fig. 1, the selection of σ_0 and α for large values of $\Delta\bar{h}_m$ will cause no special difficulties while the approximation of φ for small $\Delta\bar{h}_m$ takes on a definite arbitrariness. It should be noted that computations of the electrical conductivity of argon, performed by various authors [9, 10], diverge noticeably at temperatures of 3000-10,000°K; the electrical conductivity at these temperatures in real heaters is apparently subject to the influence of impurities and the stream turbulence. Hence, on the basis of tests comparing the theoretical and experimental volt–ampere characteristics of an arc of fixed length in argon [4], let us take the following approximation for $\varphi(\Delta\bar{h}_m)$ (curve 2 in Fig. 1):

$$\varphi = \begin{cases} 8.1\Delta\bar{h}_m^{1,2} & \text{for} \quad \Delta\bar{h}_m \leqslant 90; \\ 120\Delta\bar{h}_m^{0,6} & \text{for} \quad \Delta\bar{h}_m > 90. \end{cases}$$

It differs from that assumed in previous papers by the values of σ_0 and α for $\Delta\bar{h}_m \le 90$ (earlier $\sigma_0 = 0.54$; $\alpha = 1.8$).

Let us now write the equation for the total current taking account of (3) in the form

$$\frac{dU}{d\xi} = -\frac{20}{\pi\sigma_0\Delta\bar{h}_m^{\alpha+0,5}\Psi(\xi)}. \tag{5}$$

Substituting $\Delta\bar{h}_m$ from (4) into (5), we obtain an equation with separable variables, the integral of which is:

$$U = [F(\xi)]^{\frac{1}{\alpha+1,5}} I^{-\frac{\alpha+0,5}{\alpha+1,5}}; \tag{6}$$

$$F(\xi) = \frac{6.37(\alpha+1,5)}{\sigma_0}\int_{\xi}^{\infty}[A(\xi)]^{\alpha+0,5}[\Psi(\xi)]^{-1}d\xi. \tag{7}$$

Let us integrate (3)

$$\bar{x} = X(\xi) I^{-\frac{0,5}{\alpha+1,5}}; \tag{8}$$

$$X(\xi) = 10\int_{\xi}^{\infty}\sqrt{A(\xi)[F(\xi)]^{-\frac{1}{\alpha+1,5}}}\,\xi^{-2}d\xi. \tag{9}$$

TABLE 1. Fundamental Parameters of Experimentally Investigated Heaters

Literature source, notation in Fig. 2	$C\cdot10^{-3}$ kg/sec	d, mm	l, mm	$\bar{l}$	$I, \Omega^{-1}\cdot m^{-1}$	Re_{wd}	Q
[11], a	1,5—5	25	300	24	$5\cdot10^1$—$7\cdot10^3$	$(0,3—1)\cdot10^4$	$(1,6—5,3)\cdot10^{-3}$
[12], b	1,25—3,4	18	144	16	$2,3\cdot10^4$—$6,3\cdot10^4$	$(0,44—1,2)\cdot10^4$	$(0,9—2,4)\cdot10^{-3}$
[13], c	0,346—2,1	8	36	9	$7\cdot10^2$—$9\cdot10^4$	$(0,28—1,7)\cdot10^4$	$(1,25—7,6)\cdot10^{-4}$

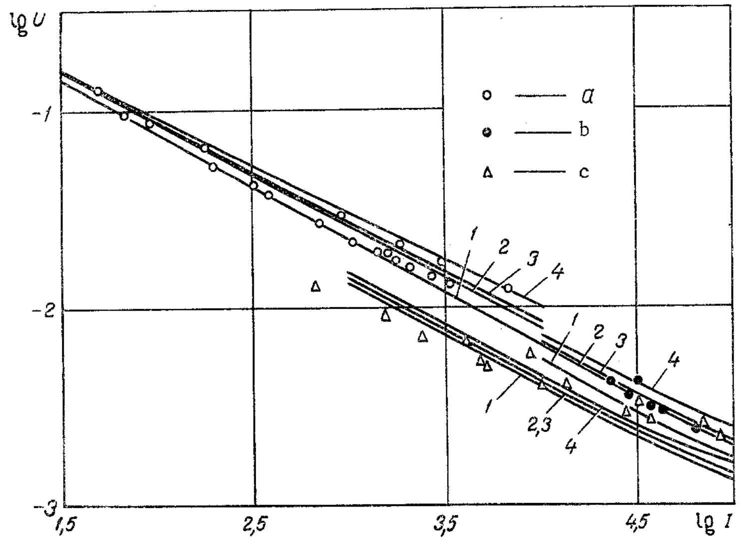

Fig. 2. Generalized volt–ampere characteristics: a-c) see Table 1. U, $\Omega \cdot m$; I, $\Omega^{-1} \cdot m^{-1}$.

Equations (6) and (8) define the function $U(\bar{x})$ in parametric form. By giving the arc length $\bar{l}$ and the values of the governing parameters I, Re_{wd}, Q, the generalized volt–ampere characteristic of the arc can be computed. Since a number of rough assumptions were taken in solving the problem, it is necessary to compare the result of the computation with experimental data.

The volt–ampere characteristics of longitudinally cooled arcs of fixed length are used for the comparison [11-13]. The fundamental parameters of the heaters for which these data have been obtained are presented in Table 1.

The following values are selected for the computation on the basis of these data: $Re_{wd} = 10^4$; $Q = 5 \cdot 10^{-4}$, and $5 \cdot 10^{-3}$. Four generalized U–I arc characteristics (Fig. 2) are presented for each of the values of $\bar{l}$ indicated in Table 1: 1) neglecting the energy loss in the channel wall ($Re_{wd} = \infty$; $Q = 0$); 2) taking account only of convective heat losses ($Re_{wd} = 10^4$; $Q = 0$); 3) taking account of convective and radiant losses $Re_{wd} = 10^4$; $Q = 5 \cdot 10^{-4}$; 4) $Re_{wd} = 10^4$; $Q = 5 \cdot 10^{-3}$. The comparison between the computed and experimental volt–ampere characteristics is presented in Fig. 2 in generalized IU coordinates.

The following peculiarities in the computed characteristics can be noted. The relation between log U and log I is almost linear for $I \leq 10^4$ and constant $\bar{l}$. The slope of the generalized characteristics diminishes as the losses taken into account grow (as Re_{wd} diminishes and Q grows). For $Q = 5 \cdot 10^{-4}$ the radiation losses exert no essential influence on the arc parameters. The influence of both the convective and radiant losses will grow as the relative length of the discharge channel $\bar{l}$ increases. A change in the slope of the characteristics occurs for constant $\bar{l}$, Re_{wd}, Q in the $10^4 < I < 10^5$ range. A computation within the framework of the turbulent arc model predicts a rise of the V–i characteristic of an arc with an intraelectrode insert in an argon stream for $I > 3 \cdot 10^4$ independently of the magnitude of the energy losses. According to the computation, rising V–i characteristics can also be obtained for small values of I for a long length of the intraelectrode insert ($\bar{l} = 24$).

The change in the slope of the V–i characteristics under the effect of energy losses is shown in Fig. 3 where the experimental and theoretical V–i characteristics of an argon-arc heater with an intraelectrode insert of diameter 25 mm and length 300 mm are represented for the discharge values 1.5 and 5 g/sec [11]. The curves 1 correspond to the conditions $Re_{wd} = \infty$, $Q = 0$ (no losses); curves 2 to the conditions $Re_{wd} = 10^4$, $Q = 0$; and curve 3 to the conditions $Re_{wd} = 10^4$, $Q = 5 \cdot 10^{-3}$ (for G = 5 g/sec, the criterion is $Q \approx 10^{-3}$, hence radiation should exert no essential influence). The passage from dropping to rising V–i characteristics for all values of the discharge from 1.5-5 g/sec has been observed in experiment [11]. The theoretical characteristics rise only in the presence of essential losses because of radiation and convective heat exchange from the wall of the discharge channel. Purely rising characteristics have been obtained in [13, 14]. Their generalization in criterion form produced a stronger dependence of the generalized drop in the voltage U on the Reynolds number ($U \sim Re^{-0.27}$ in [14]) than in the turbulent model. In this connection, note the following. The total voltage drop on the arc V usually enters into the

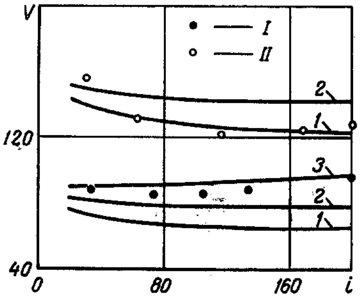

Fig. 3

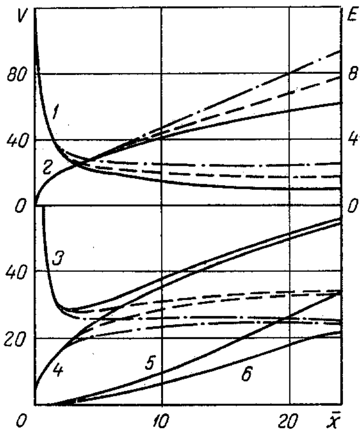

Fig. 4

Fig. 3. Comparison of theoretical and experimental volt–ampere characterististics: I) G = 0.0015; II) 0.005 kg/sec. V, V; i, A.

Fig. 4. Distributions of E (curve 1), V (2), Δh_m (3), $\Delta \bar{h}_a$ (4), Q_r/Gh_0 (5), and Q_w/Gh_0 (6) along the axis of the argon heater (i = 200 A; G = 0.0015 kg/sec); for the continuous curve, $Re_{wd} = \infty$, Q = 0; for the dotted curve, $Re_{wd} = 10^4$, Q = 0; for the dot-dash curve, $Re_{wd} = 10^4$, $Q = 5 \cdot 10^{-3}$, E(10^2 V/m), V, V.

generalization of the volt–ampere characteristics in the criterion U; however, it is physically more satisfactory for this criterion to depend on the voltage drop in the positive column of the arc, $V - V_{ac}$, since the sum of potential drops near the electrodes, V_{ac}, is practically independent of the main processes occurring in the arc column. From the relationship:

$$\frac{V - V_{\text{ac}}}{i} d = cI^{-n}$$

we obtain the dependence

$$U = \frac{Vd}{i} = cI^{-n}(1 + c_1 V_{\text{ac}} I^{n-0.5} \text{Re}^{-0.5}),$$

which explains the influence of the Reynolds number on the arc parameters in those cases when it is impossible to neglect the quantity V_{ac} as compared with the voltage drop in the arc column. This can apparently explain the stratification according to discharge (Reynolds numbers) of the data obtained in [13] for a heater with a small relative arc length ($\bar{l}$ = 9).

The solution obtained allows the distribution of the fundamental stream and discharge parameters along the axis to be computed. The results of computing the quantities E, V, Δh_a, Δh_m, Q_w/Gh_0, Q_r/Gh_0 are presented in Fig. 4 for the above-mentioned argon heater [13] for i = 200 A and G = 1.5 g/sec. The computation verifies the assumption, used in the integration, regarding the relatively small change in $\Delta \bar{h}_m$ in the major portion of the channel in the presence of energy losses from the discharge domain. The fraction of convective and radiant heat losses grows as the length of the channel increases, but it should be noted that the heat losses because of radiation are exaggerated in this case since the level of volume radiation for the theoretical value of the enthalpy ($\Delta \bar{h}_m$ = 25) is less than the magnitude of the radiation which the approximation taken (see Fig. 1) will yield. Evidently, the problem of taking radiation into account must be solved by means of the results of computing the enthalpy distribution taking only convective losses into account.

NOTATION

i is the current intensity;
V is the voltage;
E is the electrical field intensity;
ρ is the density;
u is the velocity;
h is the enthalpy;

G	is the gas discharge;
σ	is the electrical conductivity;
μ_W	is the viscosity at the wall temperature ($T_W = 300°K$);
x, r	are cylindrical coordinates;
R, d, l	are the radius, diameter, and length of the discharge channel;
b	is the radius of the mixing zone;
$\bar{x} = x/R$; $\bar{l} = l/R$; $\eta = r$; $\xi = R/b$	are dimensionless coordinates;
h_0	is the gas enthalpy at the entrance (for $T_0 = 300°K$);
q_0, σ_0, α	are coefficients of the approximating functions;
Q_W, Q_r	are the convective and radiant heat fluxes;
q_W, q_r	are the convective and radiant specific heat fluxes;
$I = i^2/Gh_0d$	is the energy criterion;
$U = Vd/i$	is the generalized voltage;
$Re_{Wd} = 4G/\pi\mu_W d$	is the Reynolds number;
$Q = q_0d^3/Gh_0$	is a radiation parameter;
$\Delta\bar{h}_a = (h_a - h_0)/h_0$	is the dimensionless excess mean-mass enthalpy
$\Delta\bar{h}_m = (h_m - h_0)/h_0$	is the dimensionless excess enthalpy on the axis.

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